Measures of Central Tendency

We have seen that the frequency distribution summaries the data. But sometimes there is need for further condensation. For this purpose we need averages. Average reduces entire data to a single number. Thus average is a single representative value which summarizes the data. This also facilitates comparison of two or more data , expressed in same units. E.g. comparison of performance of two or more sets of students , comparison of production of a company at two different time periods etc.

These Averages generally lie in the central part of the distribution , individual values of the variable cluster around it. Hence Averages are called measures of Central Tendency.

Arithmetic Mean :

Arithmetic mean is the sum of all observations divided by the number of observations.

<u>Without frequency</u>

If variable X takes values x_1 , x_2 , x_3.....x_n , then arithmetic mean is defined as

$$\bar{x} = \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots + x_n}{n}$$

$$\bar{X} = \frac{\sum x}{n}$$

<u>With frequency</u>

If variable X takes values x_1 , x_2 , x_3 ..... x_n with frequencies f_1 , f_2 , f_3 ..... f_n then arithmetic mean is defined as

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 \ldots\ldots\ldots + f_n x_n}{f_1 + f_2 + f_3 \ldots\ldots\ldots + f_n}$$

$$\bar{X} = \frac{\sum fx}{\sum f}$$

Q.1 Given below is the distribution of ages of students in a class. Calculate the mean age of the class.

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{948}{50}$$

$$= 18.96$$

Q.2 The following is the distribution of marks of 50 students in a class. Calculate the mean marks.

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{1240}{50}$$

$$= 24.8$$

Q.3 Calculate the arithmetic mean for the following data representing monthly salary of a group of employees.

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{136500}{150}$$

$$= 910$$

Q.4 The following data gives consumption of electricity. Find the average consumption.

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{137400}{170}$$

$$= 808.24$$

Combined Arithmetic Mean:

If we have two groups of n_1& n_2 observations , with means $\overline{x_1}$ and $\overline{x_2}$ respectively , the mean of combined group is given by

$$\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$$

The result can be extended to more than two groups . For three groups of n_1 ,n_2 & n_3 observations with means $\overline{x_1}$, $\overline{x_2}$ and $\overline{x_3}$ respectively , the mean of combined group is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1+n_2+n_3}$$

Q.1 In an establishment there are 90 male and 10 female workers with an average salary of Rs. 1000 and Rs. 800 respectively. Calculate arithmetic mean of monthly salaries of all employees taken together.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\bar{x} = \frac{90 \times 1000 \; + \; 10 \times 800}{90+10}$$

$$\bar{x} = \frac{90000+8000}{100}$$

$$\bar{x} = \frac{98000}{100} = 980$$

Q.2 If the average marks in Accountancy of 45 boys and 70 girls from a class are 55 and 63 respectively. Calculate average marks of the entire class.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\bar{x} = \frac{45 \times 55 \; + \; 70 \times 63}{45+70}$$

$$\bar{x} = \frac{2475+4410}{115}$$

$$\bar{x} = \frac{6885}{115} = 59.86$$

Q.3 In a health club there are 100 members 50 men , 40 women & 10 children. Their average weights are 60 Kg., 48 Kg. and 32 Kg. respectively. Calculate average weight of all members of the club.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$\bar{x} = \frac{50\times 60 \ + 40 \times 48 + 10 \times 32}{50+40+10}$$

$$\bar{x} = \frac{3000+1920+320}{100}$$

$$\bar{x} = \frac{5240}{100} = 52.4$$

Q.4 The mean scores of a group of students in a test was 52. The brightest 20% of them secured mean score of 80 and dullest 25% a mean score of 31. Calculate the mean score of remaining group.

$$\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2+n_3\bar{x}_3}{n_1+n_2+n_3}$$

$$52 = \frac{20\times 80 \ + 55 \times \overline{x_2} + 25 \times 31}{20+55+25}$$

$$52 = \frac{1600 + 55 \times \overline{x_2} + 775}{100}$$

$$5200 = 1600 + 55 \times \overline{x_2} + 775$$

$$55 \times \overline{x_2} = 5200 - 2375$$

$$= 2825$$

$$\overline{x_2} = \frac{2825}{55} = 51.36$$

Note: It is given that there are 20% bright and 25% dull students.

Since the percentages are given , they should add up to 100.

Hence percentage of remaining students is 100-(20+25)=55

$n_2 = 55$

Q. 5 A travelling salesman for the first part of the year accounted for an average sale of Rs. 1,00,000 per month. During the remaining part of the year , he improved his performance by recording an average sale of Rs. 1,24,000 per month. However , his average sale for the entire year was Rs. 1,20,000 per month. Find number of months in two parts of the year.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$1{,}20{,}000 = \frac{a \times 1{,}00{,}000 \; + \; (12-a) \times 1{,}24{,}000}{a+(12-a)}$$

Opening the brackets, we get

$$1{,}20{,}000 = \frac{1{,}00{,}000\, a + 14{,}88{,}000 - a \times 1{,}24{,}000}{12}$$

Cross multiplying, we get

$$14{,}40{,}000 = 14{,}88{,}000 - 24{,}000\, a$$

$$24{,}000\, a = 48{,}000$$

$$a = \frac{48{,}000}{24{,}000}$$

$$a = 2$$

2 months and 10 months

Note: It is given that the year is divided into 2 parts.
So total number of months should add up to 12.
If we assume there are a months in the first part , second part will contain 12-a months.
n_1 = a and n_2 = 12-a

Q. 6 Average daily pocket money of boys is Rs. 120 and that of girls is Rs. 100. If arithmetic mean of daily pocket money of boys and girls taken together is Rs. 112. Find the ratio of boys and girls in the class.

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$112 = \frac{120 \times n_1 + 100 \times n_2}{n_1 + n_2}$$

$$112n_1 + 112\, n_2 = 120 \times n_1 + 100 \times n_2$$

$$112n_1 - 120\ n_1 = = \quad 100\ n_2 - 112\, n_2$$

$$-8\ n_1 = -12\, n_2$$

$$\frac{n_1}{n_2} = \frac{12}{8}$$

$$= \frac{3}{2}$$

Weighted Arithmetic Mean:

If x_1, x_2 ,x_3 ..... x_n values of variable X are assigned weights w_1 ,w_2,w_3.....w_n respectively , then the weighted arithmetic mean is given by

$$\overline{x_w} = \frac{w_1x_1+w_2x_2+\cdots\ldots\ldots+w_nx_n}{w_1+w_2+\cdots\ldots+w_n}$$

Q1. A candidate appeared for an examination and scored 45 marks in Physics , 52 in Chemistry , 40 in Mathematics and 55 in English. If the subjects are given weights as 3,2,3,2 respectively , calculate the weighted arithmetic mean.

$$\overline{x_w} = \frac{w_1x_1+w_2x_2+w_3x_3+w_4x_4}{w_1+w_2+w_3+w_4}$$

$$= \frac{3\times45+2\times52+3\times40+2\times55}{3+2+3+2}$$

$$= \frac{135+104+120+110}{20} = \frac{469}{10} = 46.9$$

Q2. A student scores 55 marks in theory , 95 marks in practical and 25 marks for class work in a subject. Another student scores75 , 78 and 20 marks for these respectively. If theory , practical and class work are assigned weights 5,3 and 2 respectively. Find better student of the two.

Student A $\quad \overline{x_w} = \frac{w_1x_1+w_2x_2+w_3x_3}{w_1+w_2+w_3}$

$= \frac{5\times55+3\times95+2\times25}{5+3+2}$

$= \frac{610}{10} = 61$

Student B $\quad \overline{x_w} = \frac{w_1x_1+w_2x_2+w_3x_3}{w_1+w_2+w_3}$

$= \frac{5\times75+3\times78+2\times20}{5+3+2}$

$= \frac{649}{10} = 64.9$

Student B is a better student of the two

Q3. Following is the scores of 5 candidates participated in a selection procedure for a post of manager. Select the candidate on the basis of score.

weights / candidate	Written Test 5	Interview 3	Group Discussion 2
Abhijeet K.	45	20	15
Soumya Patil	30	21	16
Kiran Shah	48	18	10
Jitesh Haria	39	19	14
Yash Raj	47	22	13

$$\overline{x_w} = \frac{w_1x_1+w_2x_2+w_3x_3}{w_1+w_2+w_3}$$

Abhijeet K. $= \frac{5\times45+3\times20+2\times15}{5+3+2} = \frac{315}{10} = 31.5$

Soumya Patil. $= \frac{5\times30+3\times21+2\times16}{5+3+2} = \frac{245}{10} = 24.5$

Kiran Shah $= \frac{5\times48+3\times18+2\times10}{5+3+2} = \frac{314}{10} = 31.4$

Jitesh Haria $= \frac{5\times39+3\times19+2\times14}{5+3+2} = \frac{280}{10} = 28.0$

Yash Raj $= \frac{5\times47+3\times22+2\times13}{5+3+2} = \frac{327}{10} = 32.7$

Yash Raj is the best candidate.

Calculation of Arithmetic Mean on algebraic manipulation:

Q1. The mean output of a group of 50 workers was found to be 240 articles. Later it was discovered that the output of one worker was wrongly recorded as 320 instead of 230. Find the correct mean value.

$\bar{X} = \frac{\sum x}{n}$

$240 = \frac{\sum x}{50}$

$\sum x$ = 240 X 50

= 12000

Correct $\sum x$ = Wrong $\sum x$ – wrong observation + correct observation

= 12000 – 320 + 230

= 11910

Correct $\bar{X} = \frac{correct\ \sum x}{n} = \frac{11910}{50} = 238.2$

Q2. The mean marks of a group of 100 students was found to be 42. Later it was discovered that the marks of two students was wrongly recorded as 53 and 63 instead of 83 and 36. Find the correct mean value.

$$\bar{X} = \frac{\sum x}{n}$$

$$42 = \frac{\sum x}{100}$$

$$\sum x = 42 \text{ X } 100$$

$$= 4200$$

Correct $\sum x$ = Wrong $\sum x$ – wrong observation + correct observation

$$= 4200 - (53+63) + (83+36)$$

$$= 4203$$

Correct $\bar{X} = \frac{correct\ \sum x}{n} = \frac{4203}{100} = 42.03$

Q3. The mean a group of observations was found to be 40 When two observations 50 and 54 were added to the group, the mean rose to 42. Find the number of observations in the group initially.

Old data

$$\bar{X} = \frac{\sum x}{n}$$

$$40 = \frac{\sum x}{n}$$

$$\sum x = 40\ n$$

$$= 40\ n$$

New Data

$$\bar{X} = \frac{\sum x}{n}$$

$$42 = \frac{\sum x + 50 + 54}{n + 2}$$

Cross multiplying, we get

$$\sum x + 50 + 54 = 42\ n + 84$$

$$40\ n + 50 + 54 = 42\ n + 84$$

$$50 + 54 - 84 = 42n - 40n$$

$$2n = 20$$

$$n = 10$$

Properties of Arithmetic Mean

1. Sum of deviations of observations from mean is zero
   Deviation means difference
   Deviation from arithmetic mean will be x- $\bar{x}$

x	1	2	3	4	5
x- $\bar{x}$	-2	-1	0	1	2

$\bar{X} = \frac{\sum x}{n} = \frac{15}{5} = 3$

2. If $\bar{x}$ is known , sum of observations can be found.

$\bar{X} = \frac{\sum x}{n}$

$\sum x$ = n x $\bar{x}$

3. Sum of squares of deviations of observations from mean is less than that from any other number A.

$\sum (x - \bar{x})^2 < \sum (x - A)^2$

x	1	2	3	4	5	Total
x- $\bar{x}$	-2	-1	0	1	2	
$(x - \bar{x})^2$	4	1	0	1	4	10
X - 4	-3	-2	-1	0	1	
$(x - 4)^2$	9	4	1	0	1	15

$\sum (x - \bar{x})^2 < \sum (x - 4)^2$

4. If arithmetic mean of two groups of n_1 & $n_2$observations are $\bar{x}_1$ and $\bar{x}_2$ respectively , then arithmetic mean of the

   Combined group is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

If each of the observations is increased or decreased by any constant say M , then $\bar{x}$ also increases or decreases by M.

If each of the observations is multiplied or divided by any constant say M , then $\bar{x}$ also is multiplied or divided by M.

						Total	**Mean**
x	**1**	**2**	**3**	**4**	**5**	**15**	**3**
x-2	-1	0	1	2	3	5	1 = 3-2
X+1	2	3	4	5	6	20	4 =3+1
X*3	3	6	9	12	15	45	9= 3*3
x/2	0.5	1	1.5	2	2.5	7.5	1.5 = 3/2